

s is the order of the S diffraction efficiency peak (1, 2, 3, ...) and p is the order of the P diffraction efficiency peak (1, 2, 3, ...)

θ is the angle between the incident beam and the Bragg planes inside the medium
(A Bragg plane is a plane of maximum refractive index in the medium)

2θ is the angle between the incident beam and the diffracted beam inside the medium

The remaining parameters are defined following Eq. (1)

From Figure 2 it can be seen that $\alpha + \beta = 2\theta$, so that Equation (4) can be solved for β , the internal angle of diffraction, to yield:

$$(6) \quad \underline{\beta = a \cos\left(\frac{2p-1}{2s-1}\right) - \alpha}$$

For example, at the first possible coincidence of the two diffraction peaks $p = 1$ and $s = 2$.

Then $\cos(2\theta) = \frac{1}{3}$ and $2\theta = 70.53 \text{ deg.}$ and $\theta = 35.26 \text{ deg.}$

Note that coincidence of the sth peak of the S diffraction efficiency curve and the pth peak of the P diffraction efficiency curve will also occur when the following equation is satisfied:

$$(6) \quad \cos 2(90 - \theta) = \frac{2p-1}{2s-1}$$

$$(7) \quad \underline{\beta = 180 - a \cos\left(\frac{2p-1}{2s-1}\right) - \alpha}$$

That is, the S and P efficiency peaks will coincide when the angle between the incident beam and the Bragg planes inside the medium is either θ or $90 - \theta$. In other words, the two angles will lie equally to either side of the zero-efficiency-P angle of 45 degrees.

In the example above, where $p = 1$ and $s = 2$, we have for the second angle:

$$2\theta = 109.47 \text{ deg.} \text{ and } \theta = 54.74 \text{ deg.}$$

Since, in this case, the second angle is larger than the first angle, the dispersion will be greater and the effective thickness, T, will be less (for a given index modulation, Δn).

Clean version of the amended specification page follows:

$$(5) \Delta n = \frac{\lambda}{T} \frac{2s-1}{2} \sqrt{C_R C_S} = \frac{\lambda}{T} \left(\frac{2s-1}{2} \right) \sqrt{(\cos \alpha) \left(\cos \alpha - \frac{\lambda}{nd} \tan \left(\frac{\beta - \alpha}{2} \right) \right)}$$

where

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(A Bragg plane is a plane of maximum refractive index in the medium)

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The remaining parameters are defined following Eq. (1)

From Figure 2 it can be seen that $\alpha + \beta = 2\theta$, so that Equation (4) can be solved for β , the internal angle of diffraction, to yield:

$$(6) \beta = a \cos \left(\frac{2p-1}{2s-1} \right) - \alpha$$

Note that coincidence of the sth peak of the S diffraction efficiency curve and the pth peak of the P diffraction efficiency curve will also occur when the following equation is satisfied:

$$(7) \beta = 180 - a \cos \left(\frac{2p-1}{2s-1} \right) - \alpha$$

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The second angle will generally exceed the internal angle of total internal reflection (TIR) if the substrate is parallel to the VPG medium and the external medium is air. This problem can be overcome by using a dual-prism grism design such as that shown in Fig. 11. This type of design allows the angles of incidence and diffraction inside the medium to exceed the normal TIR angle.